Washington University in St. Louis

Nested Vector-Sensor Array Processing via Tensor Modeling [1]

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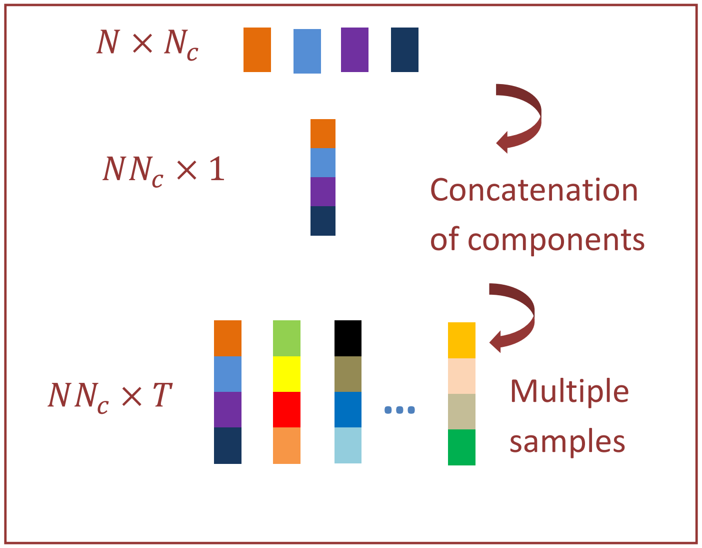
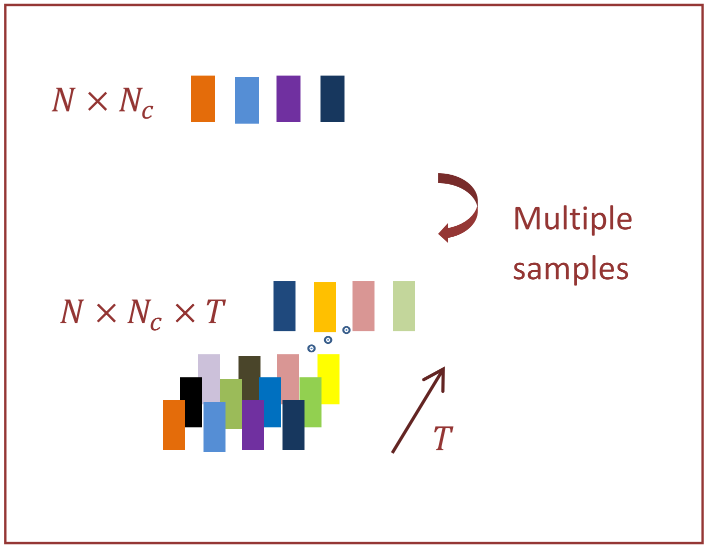
Background

- A vector sensor employs multiple (N_c) sensor components. For acoustic: $N_c = 4$; for electromagnetic: $N_c = 6$.
- Two strategies: matrix-based scheme (left) and tensor-based scheme (right).





N: Sensors

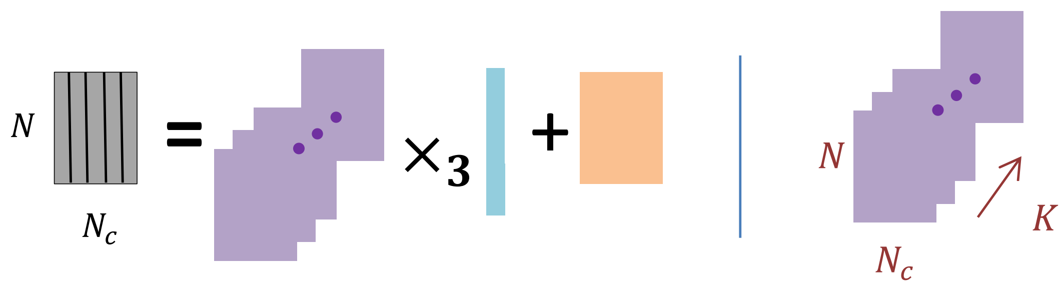
T: Snapshots

N_c: Components

- Tensor-based scheme conserves the multidimensional structures of the data, and provides better performance than the matrix-based scheme.
- Our goal: Apply the nested-array strategy to vector-sensor arrays via tensor modeling.
- Challenge: Multidimensional operation.

Signal Model

- Consider a two-level nested linear array with N vector sensors. The output of each vector sensor is an N_c-dimensional vector.
- Assume K far-field sources from directions $\{(\phi_k, \theta_k),\ k = 1, \ldots, K\}$, where ϕ_k and θ_k represent the azimuth and elevation angles
- Then, we obtain the tensor measurement model:



$$\boldsymbol{Y}(t) = \boldsymbol{\mathcal{A}} \times_3 \boldsymbol{x}(t) + \boldsymbol{E}(t), \tag{1}$$

- $\boldsymbol{Y}(t)$: $N \times N_c$ measurement matrix at time t
- $\boldsymbol{\mathcal{A}}$: $N \times N_c \times K$ array manifold tensor
- $\boldsymbol{x}(t)$: $K \times 1$ source signals
- $\boldsymbol{E}(t)$: $N \times N_c$ measurement noise at time t

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$\boldsymbol{\mathcal{A}} \times_3 \boldsymbol{\mathcal{B}}$: Mode-3 product of $\boldsymbol{\mathcal{A}} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ and $\boldsymbol{\mathcal{B}} \in \mathbb{C}^{J_1 \times J_2 \times I_3}$, defined as $(\boldsymbol{\mathcal{A}} \times_3 \boldsymbol{\mathcal{B}})_{i_1 i_2 j_1 j_2} = \sum_{i_3} a_{i_1 i_2 i_3} b_{j_1 j_2 i_3}$

Signal Model (Cont.)

- $a_{i,j,k} = (\boldsymbol{A}_k)_{i,j}$, where $\boldsymbol{A}_k = \mathrm{d}_k \boldsymbol{p}_k^T$
- $\boldsymbol{d}_k = [e^{j2\pi \boldsymbol{u}_k^T \boldsymbol{r}_1/\lambda}, \ldots, e^{j2\pi \boldsymbol{u}_k^T \boldsymbol{r}_N/\lambda}]^T$ is the phase delay vector.
- $\boldsymbol{u}_k = [\cos\phi_k \cos\theta_k, \sin\phi_k \cos\theta_k, \sin\theta_k]^T$ is the unit vector at the sensor pointing towards the kth signal.
- $\boldsymbol{p}_k$ is the steering vector of a single vector sensor located at the origin.
- Based on (1), we get the $N \times N_c \times N \times N_c$ interspectral tensor:

$$\boldsymbol{\mathcal{R}} = \mathrm{E}[\boldsymbol{Y} \circ \boldsymbol{Y}^*] = \boldsymbol{\mathcal{A}} \times_3 \boldsymbol{R}_{\boldsymbol{x}} \dot{\times}_3 \boldsymbol{\mathcal{A}}^* + \mathrm{E}[\boldsymbol{E} \circ \boldsymbol{E}^*], \tag{2}$$

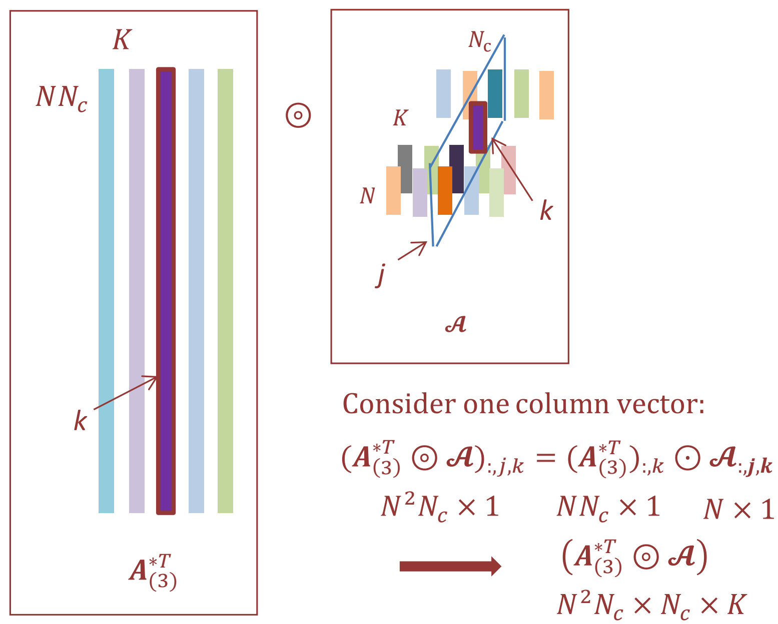
  which is a tensor version of the covariance matrix in the scalar case.
- We apply mode-2 matricization to $\boldsymbol{\mathcal{R}}$, and obtain

$$\boldsymbol{Z} \triangleq \boldsymbol{R}_{(2)}^T = (\boldsymbol{A}_{(3)}^{*T} \circledcirc \boldsymbol{\mathcal{A}}) \times_3 \boldsymbol{s} + \sigma_e^2 \overrightarrow{\boldsymbol{I}}, \tag{3}$$

  - $\boldsymbol{Z}$ is an $N_c N^2 \times N_c$ matrix
  - $\boldsymbol{s} = [\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2]^T$
  - $\overrightarrow{\boldsymbol{I}} = \mathrm{blkdiag}(\overrightarrow{\mathbf{1}}, \ldots, \overrightarrow{\mathbf{1}})$, where $\overrightarrow{\mathbf{1}} = [\boldsymbol{e}_1^T, \boldsymbol{e}_2^T, \ldots, \boldsymbol{e}_N^T]^T$

Signal Model (Cont.)

Generalized Kahtri-Rao product $\circledcirc$



$\mathcal{A}\dot{\times}_3\mathcal{B}$: Mode-3 inner product of $\mathcal{A} \in \mathbb{C}^{I_1\times I_2\times I_3}$ and $\mathcal{B} \in \mathbb{C}^{J_1\times J_2\times I_3}$, defined as $(\mathcal{A}\dot{\times}_3\mathcal{B})_{i_1i_2j_1j_2} = \sum_{i_3} a_{i_1i_2i_3}b_{j_1j_2i_3}$

$\boldsymbol{A} \circledcirc \mathcal{B}$: Generalized Khatri-Rao product of $\boldsymbol{A} \in \mathbb{C}^{J\times I_3}$ and $\mathcal{B} \in \mathbb{C}^{I_1\times I_2\times I_3}$, with dimension $I_1J \times I_2 \times I_3$, defined as $(\boldsymbol{A} \circledcirc \mathcal{B})_{(i_1+(j-1)I_1),i_2,i_3} = a_{j,i_3}b_{i_1,i_2,i_3}$

$\boldsymbol{A} \circ \boldsymbol{B}$: Outer product of $\boldsymbol{A} \in \mathbb{C}^{I_1\times I_2}$ and $\boldsymbol{B} \in \mathbb{C}^{J_1\times J_2}$, defined as $(\boldsymbol{A} \circ \boldsymbol{B})_{i_1i_2j_1j_2} = a_{i_1i_2}b_{j_1j_2}$

$\mathcal{A}_{(2)}$: mode-2 matrix unfolding of tensor $\mathcal{A} \in \mathbb{C}^{I_1\times I_2\times I_3}$, defined as $(\mathcal{A}_{(2)})_{i_2,(i_1-1)I_3+i_3} = (\mathcal{A})_{i_1i_2i_3}$

Signal Model (Cont.)

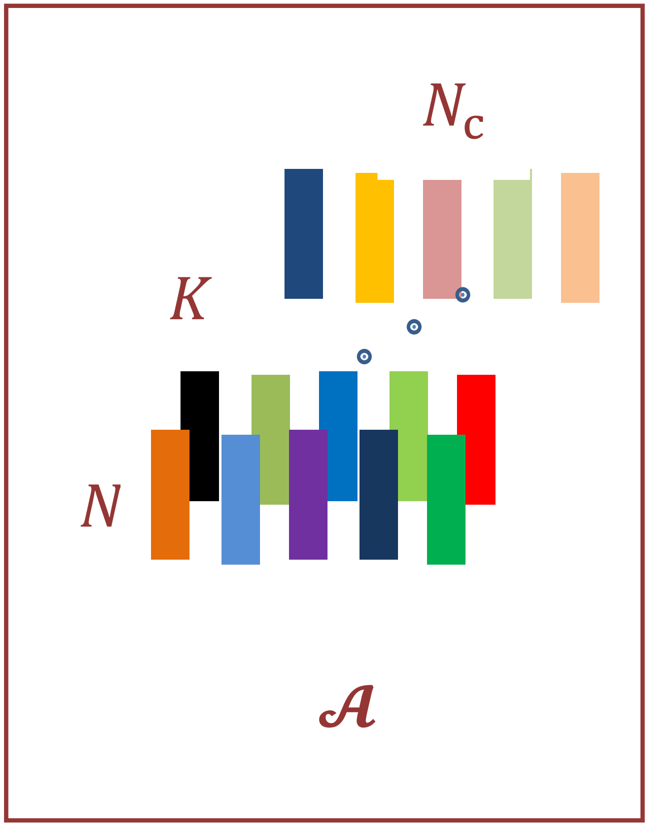
- Comparing (3) with the original signal model (1), we observe that $\boldsymbol{Z}$ in (3) behaves like a received measurement with a longer vector-sensor array whose manifold is given by $\boldsymbol{A}^{*T}_{(3)} \circledcirc \boldsymbol{\mathcal{A}}$. In (3) the equivalent source signal vector is represented by $\boldsymbol{s}$ and the noise becomes a deterministic matrix given by $\sigma_{\mathrm{e}}^2 \overrightarrow{\boldsymbol{I}}$.

- Looking at the structure of the tensor $\boldsymbol{A}^{*T}_{(3)} \circledcirc \boldsymbol{\mathcal{A}}$, we observe that there are N_{c} sets of horizontal slices, and they have same horizontal slices except for different amplitudes. Without loss of generality we will consider to use only the first set.

- Note that, although we choose only one set, we use the information from all the N_c sensor components.

Signal Model (Cont.)

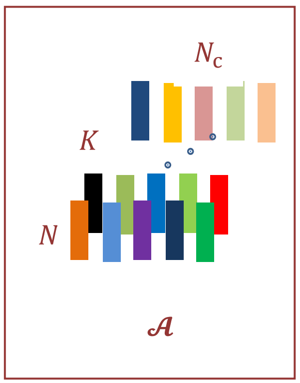
Dimension change of the manifold tensor using the nested-array strategy.



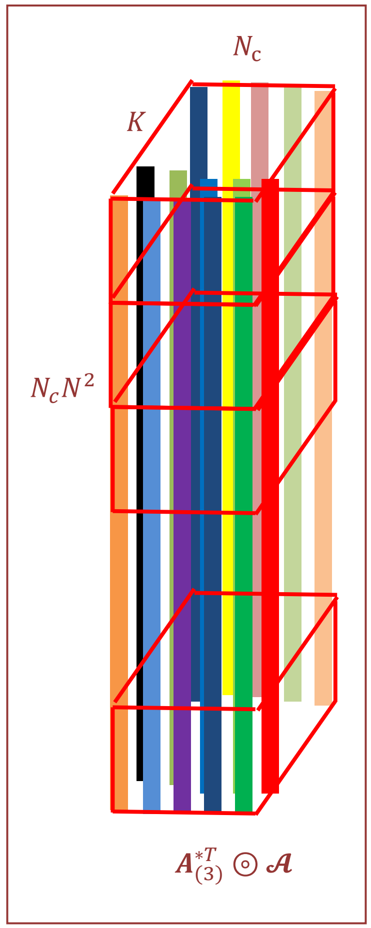
Manifold tensor $\mathcal{A}$ of the original signal model.

Signal Model (Cont.)

Dimension change of the manifold tensor using the nested-array strategy.



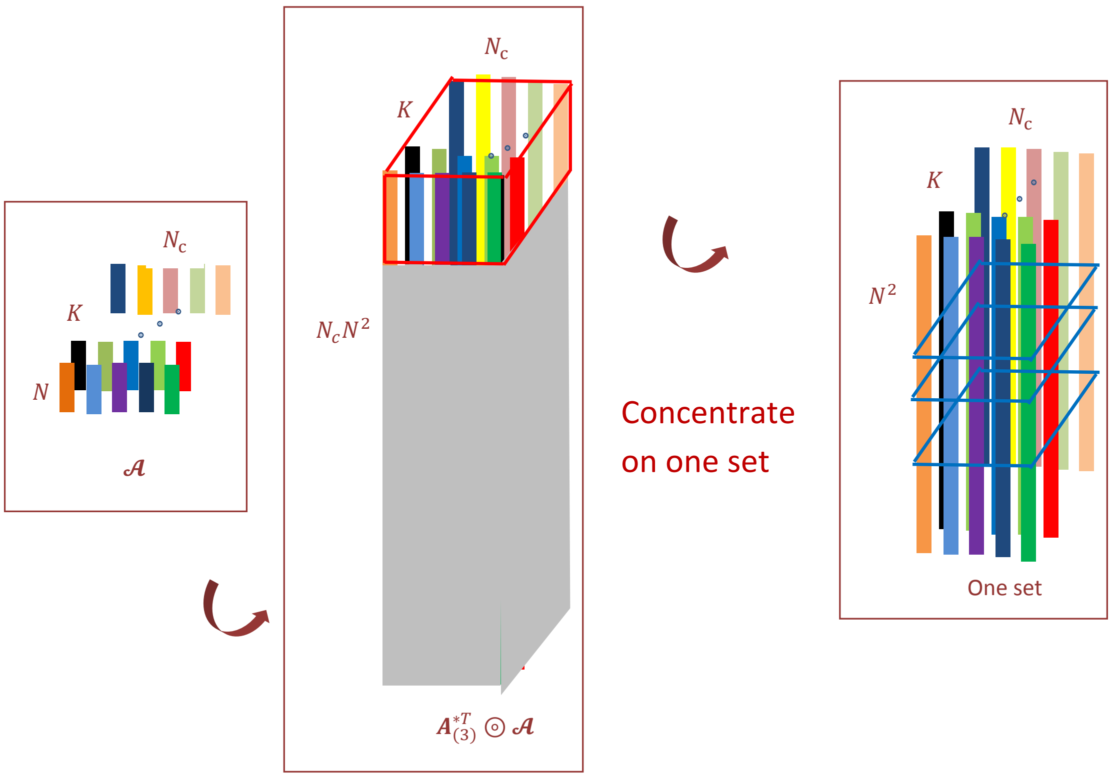
After matricizing the interspectral tensor $\mathcal{R}$



- N_c parallel sets of horizontal slices
- Different amplitudes but same information

Signal Model (Cont.)

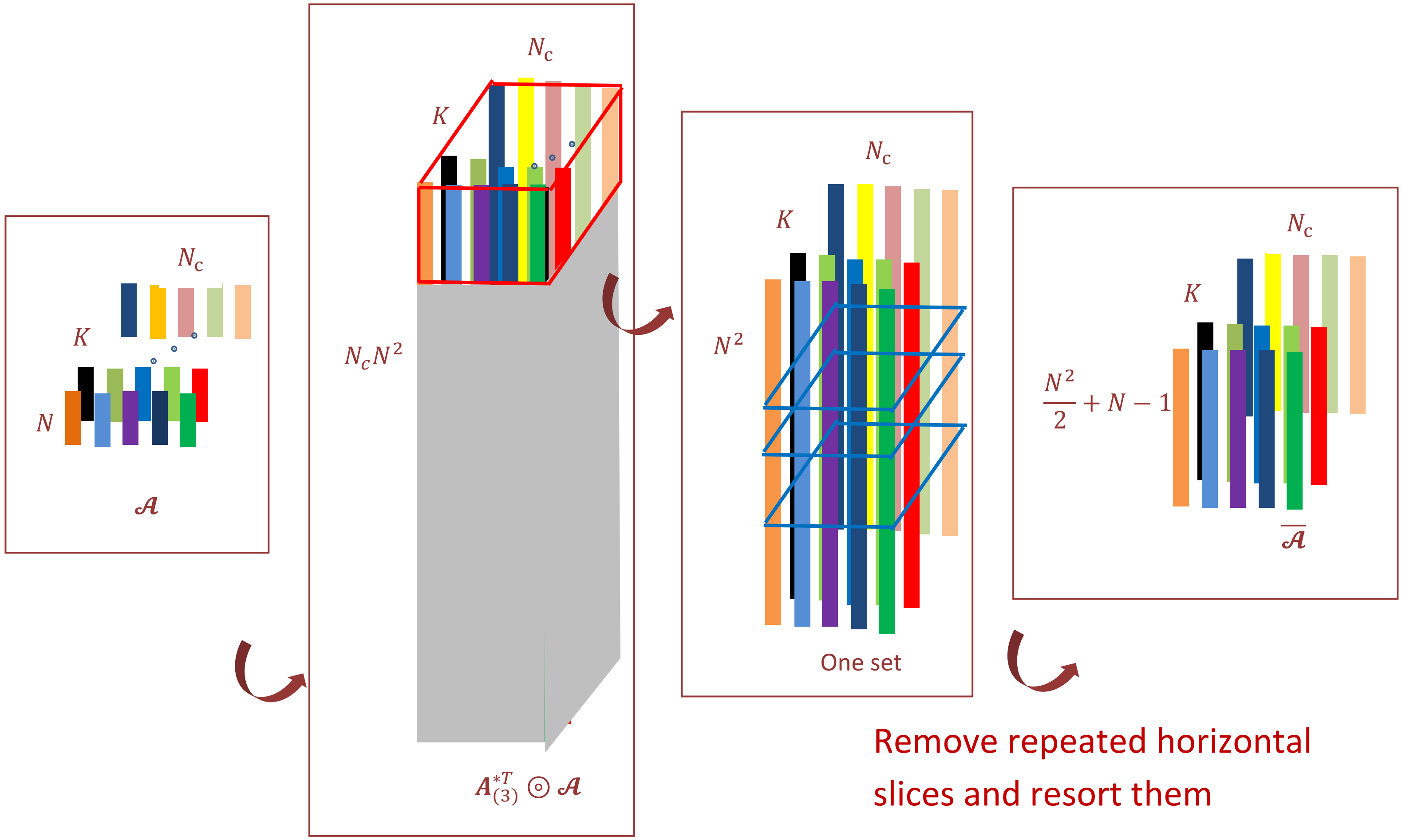
Dimension change of the manifold tensor using the nested-array strategy.



- Multiple repeated horizontal slices (matrices)

Signal Model (Cont.)

Dimension change of the manifold tensor using the nested-array strategy.



Remove repeated horizontal slices and resort them

Spatial Smoothing

- Spatial smoothing is used to exploit the increased DOFs by building up the rank of the observation tensor.
- Similar to the scalar sensor case, we remove repeated slices and sort them according to virtual sensor positions:

$$\bar{\boldsymbol{Z}} = \bar{\boldsymbol{\mathcal{A}}} \times_3 \boldsymbol{s} + \sigma_e^2 \bar{\boldsymbol{E}}.$$

- Divide these $2\bar{N} - 1$ ($\bar{N} = N^2/4 + N/2$) virtual sensors into $\bar{N}$ partially overlapping subarrays, and the lth subarray is $\bar{\boldsymbol{Z}}_l = \bar{\boldsymbol{\mathcal{A}}}_l \times_3 \boldsymbol{s} + \sigma_e^2 \bar{\boldsymbol{E}}_l$.
- Define $\boldsymbol{\mathcal{R}}_l \triangleq \bar{\boldsymbol{Z}}_l \circ \bar{\boldsymbol{Z}}_l^*$, and take the average of $\boldsymbol{\mathcal{R}}_l$ over all l: $\boldsymbol{\mathcal{T}} \triangleq \frac{1}{\bar{N}} \sum_{l=1}^{\bar{N}} \boldsymbol{\mathcal{R}}_l$.
- $\boldsymbol{\mathcal{T}}$ is the $\bar{N} \times N_c \times \bar{N} \times N_c$ spatially smoothed interspectral tensor.
- Based on $\boldsymbol{\mathcal{T}}$, the nested array with N vector sensors provides $\bar{N} - 1$ DOFs, whereas a ULA with the same number of vector sensors can provide only $N - 1$ DOFs.

Higher-Order Singular Value Decomposition (HOSVD)

- HOSVD is a higher-order generalization of the matrix singular value decomposition (SVD) [3].
- The HOSVD of tensor $\mathcal{T}$ can be written as
$$\mathcal{T} = \mathcal{K} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \times_3 \boldsymbol{U}_3 \times_4 \boldsymbol{U}_4, \tag{4}$$
where $\boldsymbol{U}_1, \boldsymbol{U}_3 \in \mathbb{C}^{\bar{N}\times\bar{N}}$, and $\boldsymbol{U}_2, \boldsymbol{U}_4 \in \mathbb{C}^{N_c\times N_c}$ are orthonormal matrices, provided by the SVD of the i-mode matricization of the tensor $\mathcal{T}$: $\mathcal{T}_{(i)} = \boldsymbol{U}_i \boldsymbol{\Lambda_i} \boldsymbol{V}_i^H$. $\mathcal{K} \in \mathbb{C}^{\bar{N}\times N_c\times\bar{N}\times N_c}$ is the core tensor.
- Since $\mathcal{T}$ is an Hermitian tensor, i.e., $t_{i_1,i_2,i_3,i_4} = t^*_{i_3,i_4,i_1,i_2}$, $\forall i_1, i_2, i_3, i_4$, the HOSVD of $\mathcal{T}$ can be written as
$$\mathcal{T} = \mathcal{K} \times_1 \boldsymbol{U}_1 \times_2 \boldsymbol{U}_2 \times_3 \boldsymbol{U}_1^* \times_4 \boldsymbol{U}_2^*. \tag{5}$$

[3] M. Boizard, G. Ginolhac, F. Pascal, S. Miron, and P. Forster, Numerical performance of a tensor MUSIC algorithm based on HOSVD for a mixture of polarized sources, in *EUSIPCO 2013*, Marrakech, Marocco, Sep. 2013.

Applications: Acoustic Vector Sensors

- For acoustic vector sensors, $N_\mathrm{c} = 4$.
- The array steering matrix can be written as $\boldsymbol{A}_k = \boldsymbol{d}_k \boldsymbol{p}_k^T$, with

$$\boldsymbol{p}_k = [1, \boldsymbol{u}_k^T]^T,$$

which is the steering vector of a single vector sensor located at the origin.

Applications: Electromagnetic Vector Sensors

- For EM vector sensors, $N_{\mathrm{c}} = 6$. Here, we consider polarized signals.

- The array steering matrix can be written as $\boldsymbol{A}_k = \boldsymbol{d}_k \boldsymbol{p}_k^T$, with $\boldsymbol{p}_k = \boldsymbol{V}_k \boldsymbol{\rho}_k$, where

$$\boldsymbol{V}_k = \begin{pmatrix} -\sin\phi_k & -\cos\phi_k \sin\theta_k \\ \cos\phi_k & -\sin\phi_k \sin\theta_k \\ 0 & \cos\theta_k \\ -\cos\phi_k \sin\theta_k & \sin\phi_k \\ -\sin\phi_k \sin\theta_k & -\cos\phi_k \\ \cos\theta_k & 0 \end{pmatrix}, \text{ and}$$

$$\boldsymbol{\rho}_k = [\cos\gamma_k \ \sin\gamma_k e^{j\eta_k}]^T.$$

- $\boldsymbol{A}_k$ is the $N \times N_c$ steering matrix of the array associated with a polarized signal coming from the direction (ϕ_k, θ_k) with polarization (γ_k, η_k), where $\gamma_k \in [0, 2\pi]$ and $\eta_k \in (-\pi, \pi]$ are polarization parameters. $\boldsymbol{V}_k$ is the steering matrix of one EM vector sensor associated with the kth signal. $\boldsymbol{\rho}_k$ is the polarization vector for the kth signal.

Acoustic Case I: MUSIC Spectrum

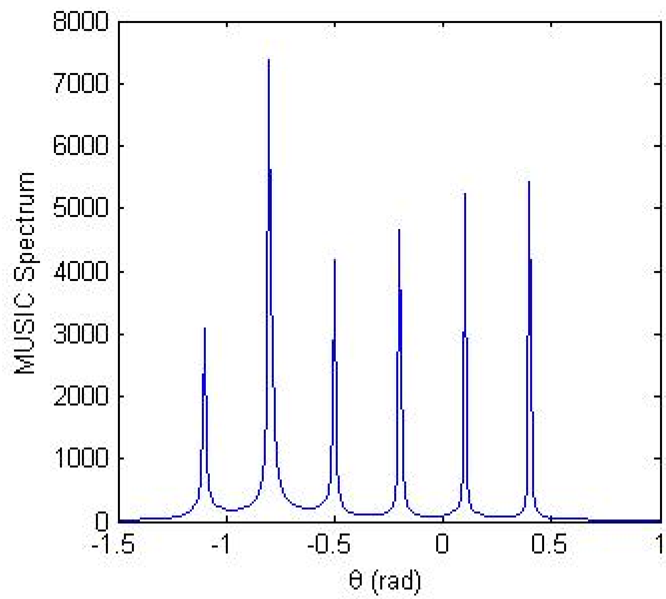


Fig. 1: MUSIC spectrum using a nested acoustic vector-sensor array with 6 sensors, as a function of elevation angle θ, $T = 1000$, $\mathrm{SNR} = 0$dB, $K = 6$ sources with the same azimuth angles.

Observation: The 2-level nested array resolves all the 6 sources.

EM Case II: DOA Estimation

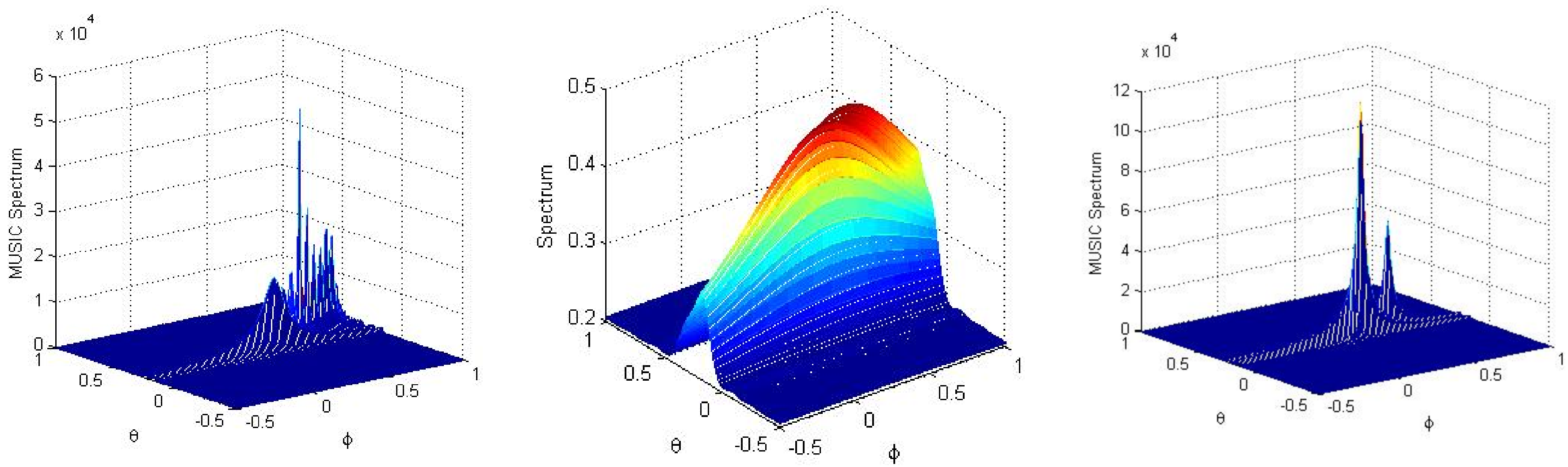


Fig. 2: MUSIC spectrum using a ULA (left: tensor-based) and a nested array (middle: matrix-based; right: tensor-based) with 6 EM vector sensors, as a function of azimuth ϕ and elevation angles θ, $T = 1000$, $\mathrm{SNR} = 21.97\mathrm{dB}$, $K = 2$.

Observation: The proposed nested vector-sensor array strategy outperforms the ULA with the same number of sensors. In addition, the tensor-based method outperforms the matrix-based method.

EM Case III: Source Number Detection

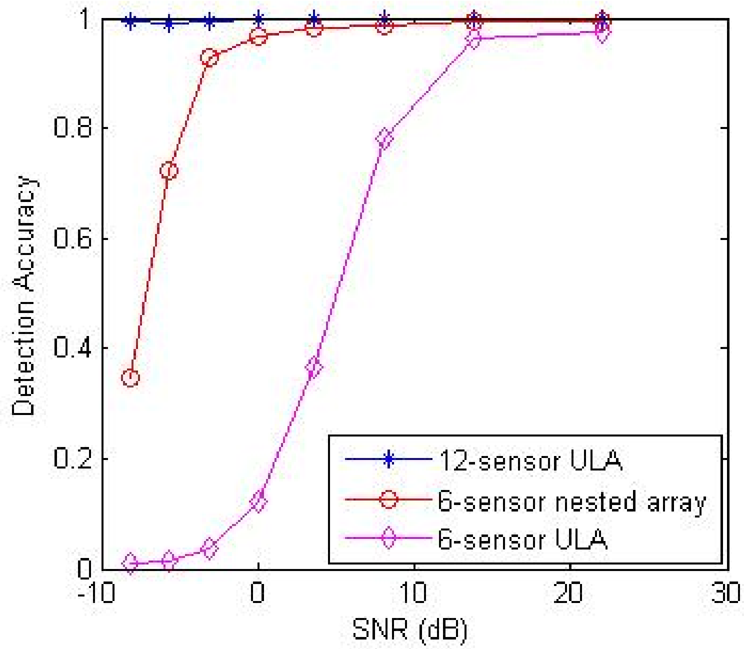


Fig. 3: Probability of detection versus SNR using a nested array with 6 EM vector sensors and ULAs with 6 and 12 EM vector sensors, $T = 1000$, $K = 2$.

Observation: We can see that the detection performance of all the three arrays improves with increasing SNRs. In addition, the nested array outperforms the corresponding ULA with same number of sensors and performs closer to the ULA with double number of sensors.

Summary

- Nested vector-sensor array processing via tensor modeling
  - We established the analytical foundation of the nested vector-sensor array by exploiting multilinear algebra
  - We constructed corresponding signal processing strategies, and verified their effectiveness through numerical examples.

Reference

- Journal Papers

1. K. Han and A. Nehorai, "Nested vector-sensor array processing via tensor modeling," to appear in *IEEE Trans. on Signal Processing*.

- Conference Papers

1. K. Han and A. Nehorai, "Direction of arrival estimation using nested vector-sensor arrays via tensor modeling," *Proc. 8th IEEE Sensor Array and Multichannel Signal Processing (SAM) Workshop*, A Coruña, Spain, Jun. 22-25, 2014 (**invited**).